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Therefore

$$a^2 + b^2 + \left( \frac{a^2 + b^2 - c^2}{2c} \right)^2 = \left( \frac{a^2 + b^2 + c^2}{2c} \right)^2,$$

whatever be the values of  $a, b, c$ .

1. Taking  $a = 1, b = 2, c = 1$ , we find

$$1^2 + 2^2 + 2^2 = 3^2,$$

which is the *smallest* rational parallelepiped.

2. Taking  $a = 2, b = 3, c = 1$ , we have

$$2^2 + 3^2 + 6^2 = 7^2,$$

which is the smallest rational parallelepiped having its edges all different.

3. Taking  $a = 1, b = 4, c = 1$ , we get

$$1^2 + 4^2 + 8^2 = 9^2.$$

4. Let  $a = 2, b = 6, c = 2$ , and we have

$$2^2 + 6^2 + 9^2 = 11^2.$$

5. If  $a = 3, b = 4, c = 1$ , we get

$$3^2 + 4^2 + 12^2 = 13^2,$$

which, on p. 269, is stated to be "the smallest rational parallelepiped."

6. If we take  $a = 8, b = 9, c = 5$ , we will get

$$8^2 + 9^2 + 12^2 = 17^2.$$

7. Taking  $a = 4, b = 8, c = 2$ , we have

$$4^2 + 8^2 + 19^2 = 21^2.$$

8. If  $a = 3, b = 4, c = 3$ , then we get

$$12^2 + 15^2 + 16^2 = 25^2.$$

And so on, there being an infinite number of parallelepipeds whose edges and solid diagonals are rational integers.

The condition  $x^2 + y^2 + z^2 = \square$  can be satisfied in many ways.—See *Mathematical Magazine*, Vol. II., No. 5 (October, 1891), pp. 71-74. Other methods are given in a forthcoming paper which will appear in the third part of No. 12, Vol. II., of the *Mathematical Magazine*.

**416A. Proposed by H. O. HANSON, East Elmhurst, N. Y.**

Find the  $n$ th term and the sum of  $n$  terms of the series obeying the relation  $u_i = u_{i-1} + 2u_{i-2}$  in terms of  $n$  and the first two terms,  $u_1$  and  $u_2$ , these two terms being arbitrary.

This problem was incorrectly numbered 416.

SOLUTION BY S. A. JOFFE, New York City.

Adding  $u_{i-1}$  to both members of the given relation, we obtain the following recurring formula:

$$u_i + u_{i-1} = 2(u_{i-1} + u_{i-2}). \quad (1)$$

If we replace  $i$  successively by  $n$ ,  $n-1$ ,  $n-2$ ,  $\dots$ , 3 and multiply the resulting  $n-2$  equations, we have, after cancellation:

$$u_n + u_{n-1} = 2^{n-2}(u_2 + u_1). \quad (2)$$

Similarly

$$u_{n-2} + u_{n-3} = 2^{n-4}(u_2 + u_1),$$

$$u_{n-4} + u_{n-5} = 2^{n-6}(u_2 + u_1),$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot,$$

the last equation of this type being  $u_2 + u_1 = u_2 + u_1$ , or  $u_3 + u_2 = 2(u_2 + u_1)$ , according as  $n$  is even or odd.

Adding all these equations, we have:

$$\text{for } n = \text{even}, \quad \sum_{i=1}^n u_i = (2^{n-2} + 2^{n-4} + 2^{n-6} + \dots + 1)(u_2 + u_1),$$

or

$$\sum_{i=1}^n u_i = \frac{1}{3}(2^n - 1)(u_2 + u_1); \quad (3)$$

and for  $n = \text{odd}$ ,

$$\sum_{i=1}^n u_i = (2^{n-2} + 2^{n-4} + 2^{n-6} + \dots + 2)(u_2 + u_1) + u_1 = \frac{2}{3}(2^{n-1} - 1)(u_2 + u_1) + u_1,$$

or

$$\sum_{i=1}^n u_i = \frac{1}{3}(2^n - 2)(u_2 + u_1) + u_1. \quad (4)$$

Formulae (3) and (4) express the sum of the first  $n$  terms, as required, in terms of  $n$  and the first two terms; the  $n$ th term  $u_n$  is found by subtracting  $\sum_{i=1}^{n-1} u_i$  from

$\sum_{i=1}^n u_i$ , the result being for  $n = \text{even}$ ,

$$\begin{aligned} u_n &= \frac{1}{3}(2^n - 1)(u_2 + u_1) - [\frac{1}{3}(2^{n-1} - 2)(u_2 + u_1) + u_1] \\ &= \frac{1}{3}(2^{n-1} + 1)(u_2 + u_1) - u_1; \end{aligned}$$

and for  $n = \text{odd}$ ,  $u_n = [\frac{1}{3}(2^n - 2)(u_2 + u_1) + u_1] - \frac{1}{3}(2^{n-1} - 1)(u_2 + u_1)$

$$= \frac{1}{3}(2^{n-1} - 1)(u_2 + u_1) + u_1.$$

The last two formulæ may be combined into one, for  $n$  in general,

$$u_n = \frac{1}{3}[2^{n-1} + (-1)^n](u_2 + u_1) + (-1)^{n-1}u_1. \quad (5)$$

Also solved by A. M. HARDING, A. H. HOLMES, HORACE OLSON, and the PROPOSER.